APPENDIX A

A.1 MATHEMATICAL DERIVATION OF GOVERNING EQUATIONS OF THE CREEP MODEL

The strain rate tensor for elasto-viscoplastic material is decomposed into elastic and viscoplastic parts as follows:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}{}^{e} + \dot{\varepsilon}_{ij}{}^{vp} \tag{A.1}$$

where *e* and *vp* respectively denote the elastic and viscoplastic components of $\dot{\varepsilon}_{kl}$. The elastic stiffness matrix is same as that in modified Cam clay (MCC) formulation (Schofield and Wroth, 1968), and hence the elastic strain rate is given by:

$$\dot{\varepsilon}_{ij}{}^{e} = D^{e}\dot{\sigma}_{ij} \tag{A.2}$$

where D^e is the elastic stress-strain matrix, $\dot{\sigma}_{ij}$ is the rate of effective stress. The prime symbol used to differentiate effective stresses from total stresses was omitted here, as all the stress quantities considered in the formulation of creep model are effective stress quantities.

Even though, zero elastic shear strains were assumed in the MCC formulation, to circumvent the difficulty of implementing the MCC model in a finite element program with this assumption, the program is allowed to calculate elastic shear strains inside the yield locus. Same technique is used in the creep model formulation as well, and to calculate the terms of D^e matrix in the elastic range, the stress dependent elastic bulk modulus *K* is calculated as:

$$K = (1+e)p/\kappa \tag{A.3}$$

where p is the mean effective stress, e is the void ratio and κ is the recompression index in the natural log scale. The shear modulus, G, is related to bulk modulus via the following expression:

$$G = \frac{3(1-2\nu)K}{2(1+\nu)} \tag{A.4}$$

where ν is the Poisson's ratio.

To incorporate the viscous nature within the classical plasticity framework two surfaces are assumed, in the present formulation, implying an associative flow rule as shown in Fig. A.1. They are, (a) Modified Cam clay elliptical surface and (b) Modified Cam clay elliptical functional surface.

Modified Cam clay elliptical functional surface is of constant Φ with the following form:

$$\bar{f} = (\bar{p} - \bar{p}_0)\bar{p} + \left(\frac{\bar{q}}{M}\right)^2 = 0 \tag{A.5}$$

where M is the slope of the critical state line, $\bar{p}_0$, which is the intersection of $\bar{f}$ with the positive p axis can be considered as a preconsolidation mean effective pressure, $\bar{q}$ is the deviator stress, $\bar{p}$ is the mean effective stress and in general three dimensional stress space $\bar{q}$ and $\bar{p}$ are defined appropriately as,

$$\bar{q} = \left\{\frac{1}{2}\left[(\bar{\sigma}_1 - \bar{\sigma}_2)^2 + (\bar{\sigma}_2 - \bar{\sigma}_3)^2 + (\bar{\sigma}_3 - \bar{\sigma}_1)^2\right]\right\}^{\frac{1}{2}} \tag{A.6}$$

$$\bar{p} = (\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)/3 \tag{A.7}$$

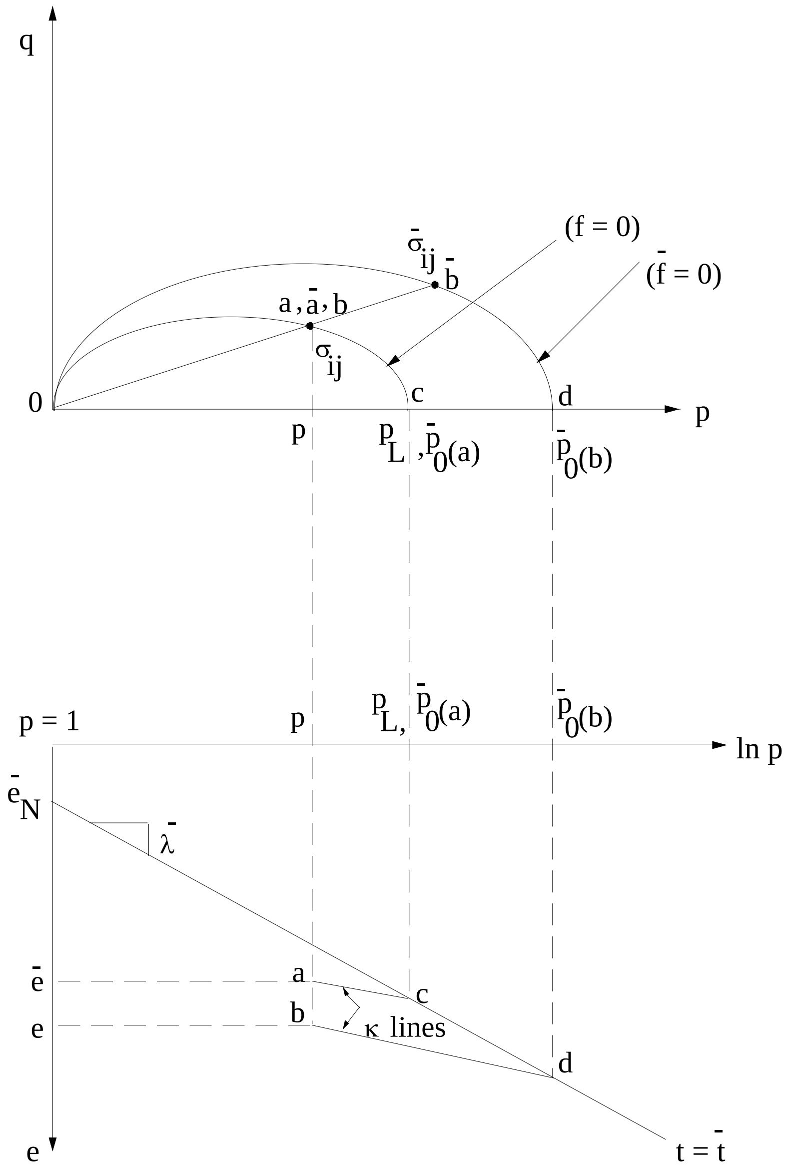

FIG. A.1 LOCATIONS OF MEAN EFFECTIVE STRESS IN e-ln p SPACE AND p-q SPACE where $\bar{\sigma}_1$, $\bar{\sigma}_2$ and $\bar{\sigma}_3$ are principal effective stresses. The Modified Cam clay elliptical surface is a surface of constant Φ and similar in shape to $\bar{f}$. This contains the current stress state. The f surface function can be obtained by replacing f, p, p_L, q for $\bar{f}$, $\bar{p}$, $\bar{p}_0$, $\bar{q}$ respectively in equation (A.5).

The stress on $\bar{f}$ surface, $\bar{\sigma}_{ij}$, is then related to the current stress state, σ_{ij}, on the f surface through the radial mapping rule with the projection centre at the origin, as illustrated in Fig. A.1. Dafalias and Hermann (1982) also used the radial mapping rule in their bounding surface formulation. The radial mapping rule gives the following relationships:

$$\bar{\sigma}_{ij} = \beta \sigma_{ij} \tag{A.8a}$$

$$\bar{p} = \beta p \tag{A.8b}$$

$$\bar{q} = \beta q \tag{A.8c}$$

where $\beta = \left(\bar{p}_0 / p_L \right)$ is a mapping parameter. Even though, very complicated expressions for β were reported in the literature for different shapes of yield surfaces (Dafalias and Hermann, 1982; Kaliakin, 1985), a relatively simple expression for β is used in the present formulation, because of similar shape of elliptical surfaces assumed for both elliptical surfaces.

The normality rule is applied on to the $\bar{f}$ surface in the present formulation. Hence, the stress strain relation for the viscoplastic strain rate for associative flow rule is,

$$\dot{\varepsilon}_{ij}^{\,vp} = \phi \frac{\partial \bar{f}}{\partial \bar{\sigma}_{ij}} \tag{A.9a}$$

$$\dot{\varepsilon}_{v}^{\,vp} = \phi \frac{\partial \bar{f}}{\partial \bar{p}} \tag{A.9b}$$

$$\dot{\varepsilon}_q^{\ vp} = \phi \frac{\partial \bar{f}}{\partial \bar{q}} \quad \text{(A.9c)}$$

where $\dot{\varepsilon}_v^{\ vp}$ is the volumetric viscoplastic strain rate and $\dot{\varepsilon}_q^{\ vp}$ is the deviatoric viscoplastic strain rate.

Creep at constant effective stress is usually defined using the secondary compression index (C_α) as follows:

$$C_\alpha = -\frac{\Delta e}{\Delta(\log t)} \quad \text{(A.10a)}$$

and

$$\alpha = {C_\alpha}/{\ln 10} \quad \text{(A.10b)}$$

where t is time and Δe is the change in void ratio during creep compression. This is illustrated in Fig. A.1. The soil sample at point 'a' sitting for an exact time $t = t$ with respect to reference time ($\bar{t}$), undergoes creep as time passes and the reference void ratio $(\bar{e})$ decreases from 'a' to 'b', whereas the apparent preconsolidation pressure increases from 'c' to 'd'. If the final void ratio at point 'b' after creeping is e, then the equation (A.10a) can be rewritten as,

$$C_\alpha = -\left(\frac{\bar{e} - e}{\log \bar{t} - \log t}\right) \quad \text{(A.11)}$$

The reference time, $\bar{t}$, is normally taken as one day. However, the reference time may be chosen from 1-D consolidation test to match the load duration used in laboratory testing. The reference void ratio, $\bar{e}$, would be obtained if the sample was normally consolidated at the same stress for reference time $\bar{t}$. Using equations (A.10b) and (A.11) the following equation can be deduced:

$$t/\bar{t} = \exp\left(\frac{\bar{e}-e}{\alpha}\right) \tag{A.12}$$

Differentiating the equation (A.12) with respect to time,

$$\frac{de}{dt} = -\frac{\alpha \exp\left((e-\bar{e})/\alpha\right)}{\bar{t}} \tag{A.13}$$

but,

$$\dot{\varepsilon}_v^{\ vp} = -\frac{de}{dt}\frac{1}{(1+\bar{e})} \tag{A.14}$$

where $\dot{\varepsilon}_v^{\ vp}$ is the volumetric viscoplastic strain rate. Comparing equations (A.13) and (A.14),

$$\dot{\varepsilon}_v^{\ vp} = \frac{\alpha}{\bar{t}(1+\bar{e})}\exp\left((e-\bar{e})/\alpha\right) \tag{A.15}$$

The following two equations can be obtained from Fig. A.1:

$$e = \bar{e}_N - \bar{\lambda}\ln \bar{p}_0 + \kappa \ln\left(\frac{\bar{p}_0}{p}\right) \tag{A.16}$$

$$\bar{e} = \bar{e}_N - \bar{\lambda}\ln p_L + \kappa \ln\left(\frac{p_L}{p}\right) \tag{A.17}$$

where $\bar{e}_N$ is the void ratio for the reference time at unit mean normal pressure on the isotropic normal consolidation line. $\bar{e}_N$ can be related to Γ parameter defined in the modified Cam clay formulation (Schofield and Wroth, 1968) via the following equation:

$$\bar{e}_N = \Gamma + \left(\bar{\lambda} - \kappa\right)\ln 2 \tag{A.18}$$

From equations (A.16) and (A.17),

$$\left(e - \bar{e}\right) = \left(\bar{\lambda} - \kappa\right)\ln\left(\frac{p_L}{\bar{p}_0}\right) \tag{A.19}$$

Substituting equation (A.19) into equation (A.15) gives,

$$\dot{\varepsilon}_v^{\ vp} = \frac{\alpha}{\ddot{t}\left(1+\bar{e}\right)}\left(\frac{p_L}{\bar{p}_0}\right)^{\left(\frac{\bar{\lambda}-\kappa}{\alpha}\right)} \tag{A.20}$$

The viscoplastic flow function Φ can therefore be determined theoretically by combining equations (A.9b) and (A.20) as follows:

$$\phi = \frac{\alpha}{\bar{t}\left(1+\bar{e}\right)}\frac{1}{\left(\partial\bar{f}/\partial\bar{p}\right)}\left(\frac{p_L}{\bar{p}_0}\right)^{\left(\frac{\bar{\lambda}-\kappa}{\alpha}\right)} \tag{A.21}$$

All the terms in equation (A.21) have been defined previously except the partial derivatives of function $\bar{f}$. For the sake of completion, the partial derivatives were obtained by differentiating the function $\bar{f}$ defined in equation (A.5), and given blow. Likewise, the partial derivatives of $\bar{p}$ and $\bar{q}$ with respect to the stress state on the $\bar{f}$ surface, $\bar{\sigma}_{ij}$, can also be obtained as follows:

$$\frac{\partial\bar{f}}{\partial\bar{p}} = 2\bar{p} - \bar{p}_0 \tag{A.22a}$$

$$\frac{\partial\bar{f}}{\partial\bar{q}} = \frac{2\bar{q}}{M^2} \tag{A.22b}$$

$$\frac{\partial \overline{p}}{\partial \overline{\sigma}_{ij}} = \frac{1}{3}\delta_{ij} \tag{A.23}$$

$$\frac{\partial \overline{q}}{\partial \overline{\sigma}_{ij}} = \frac{3}{2\overline{q}}\left(\overline{\sigma}_{ij} - \overline{p}\delta_{ij}\right) \qquad \text{for} \qquad (i = j) \tag{A.24a}$$

$$\frac{\partial \overline{q}}{\partial \overline{\sigma}_{ij}} = \frac{3}{2\overline{q}}\left(2\overline{\sigma}_{ij}\right) \qquad \text{for} \qquad (i \neq j) \tag{A.24b}$$

where δ_{ij} is the Kronecker delta. Thus, the normal at any point on $\bar{f}$ is given by,

$$\frac{\partial \bar{f}}{\partial \overline{\sigma}_{ij}} = \frac{\partial \bar{f}}{\partial \overline{p}}\frac{\partial \overline{p}}{\partial \overline{\sigma}_{ij}} + \frac{\partial \bar{f}}{\partial \overline{q}}\frac{\partial \overline{q}}{\partial \overline{\sigma}_{ij}} \tag{A.25a}$$

$$= \left(\frac{2\overline{p}}{3} - \frac{3\overline{p}}{M^2} - \frac{\overline{p}_0}{3}\right)\delta_{ij} + \frac{3}{M^2}\overline{\sigma}_{ij} \tag{A.25b}$$

Substituting these derivatives into equation (A.9a), the elasto-viscoplastic strain rate will be obtained as,

$$\dot{\varepsilon}_{ij}^{\ vp} = \left[\frac{\alpha}{\bar{t}(1+\bar{e})}\frac{1}{(2\overline{p} - \overline{p}_0)}\left(\frac{p_L}{\overline{p}_0}\right)^{\left(\frac{\bar{\lambda}-\kappa}{\alpha}\right)}\right]\left[\left(\frac{2\overline{p}}{3} - \frac{3\overline{p}}{M^2} - \frac{\overline{p}_0}{3}\right)\delta_{ij} + \frac{3}{M^2}\overline{\sigma}_{ij}\right]$$

(A.26)

A.1.1 Derivation of time dependent hardening rule

The time dependent evolution law of the $\bar{f}$ surface is depending on the current viscoplastic increment of volumetric strain, $\partial\varepsilon_v^{\ vp}$, and is accounted for via equation (A.19):

$$\left(e - \bar{e}\right) = \left(\bar{\lambda} - \kappa\right)\ln\left(\frac{p_L}{\overline{p}_0}\right) \tag{A.19 \textit{bis}}$$

Taking derivatives with respect to time of equation (A.19) and substitute $de = -d\varepsilon_v^{vp}(1+\bar{e})$, and $\frac{d\bar{e}}{dt} = -(1+\bar{e})\dot{\varepsilon}_v^{vp}$ the time dependent hardening rule can be obtained as follows:

$$\dot{\bar{p}}_0 = \frac{d\bar{p}_0}{dt} = \bar{p}_0 \exp\left(\frac{(1+\bar{e})\partial\varepsilon_v^{vp}}{(\kappa - \bar{\lambda})}\right)\left(\frac{(1+\bar{e})\partial\varepsilon_v^{vp}}{(\bar{\lambda} - \kappa)}\right) \qquad \text{(A.27)}$$

All the terms in equation (A.27) have been defined previously. It is noted from this equation that $\bar{p}_0$ changes with time dependent viscoplastic deformation and the time is implicitly represented by $\partial\varepsilon_v^{vp}$.